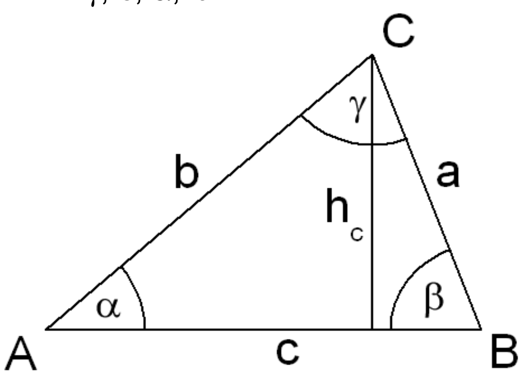
Berechnung geradlinig begrenzter Figuren – Lösungen

1. Berechne mit den gegebenen Werten des Dreiecks die fehlenden Seiten und Winkel.

Gegeben:	**Gesucht:**
a) h_c = 14,4 cm; α = 53,1°; ß = 28,1°	γ, a, b, c
b) b = 12,5 cm; h_a = 10,1 cm; α = 65,5°	γ, ß, a, c
c) h_b = 11,3 cm; h_c = 18,3 cm; a = 20 cm	γ, ß, α, b, c
d) b = 5,1 cm; h_b = 1,2 cm; γ = 18,5°	a, c, α, ß
e) a = 8 cm; h_a = 9,1 cm; c = 10 cm	ß, b, α, γ
f) a = 2,2 cm; b = 4 cm; s_b = 1,8 cm	γ, c, α, ß

Lösung Aufgabe a)

$\gamma = 180° - \alpha - ß = 98,8°$



$$\sin ß = \frac{h_c}{a}$$

$$a = \frac{h_c}{\sin ß} = 30,57 \text{ cm}$$

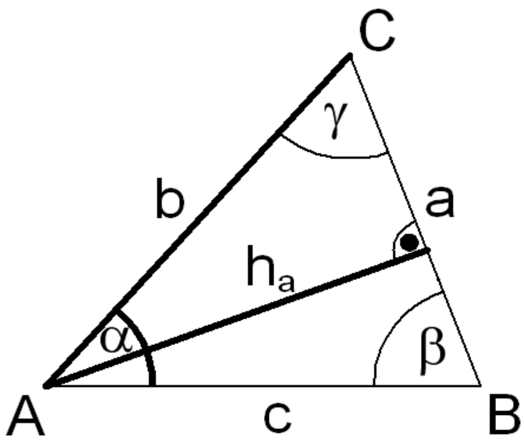
$$\sin \alpha = \frac{h_c}{b}$$

$$b = \frac{h_c}{\sin \alpha} = 18,01 \text{ cm}$$

$$\frac{a}{c} = \frac{\sin \alpha}{\sin ß}$$

$$c = \frac{a \cdot \sin \alpha}{\sin ß} = 37,78 \text{ cm}$$

Lösung Aufgabe b)



$$\sin \gamma = \frac{h_a}{b}$$

$$\gamma = 53,9°$$

$$ß = 180° - \alpha - \gamma = 60,6°$$

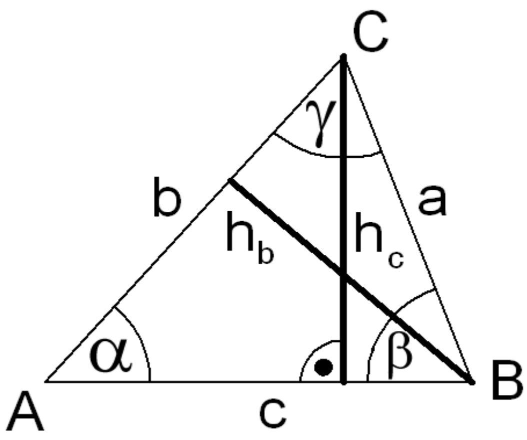
$$\frac{a}{b} = \frac{\sin \alpha}{\sin ß}$$

$$a = \frac{b \cdot \sin \alpha}{\sin ß} = 13,06 \text{ cm}$$

$$\frac{a}{c} = \frac{\sin \alpha}{\sin \gamma}$$

$$c = \frac{a \cdot \sin \gamma}{\sin \alpha} = 11,6 \text{ cm}$$

Lösung Aufgabe c)



$$\sin\gamma = \frac{h_b}{a}$$

$$\gamma = 34,4°$$

$$\sin ß = \frac{h_c}{a}$$

$$ß = 66,2°$$

$$\alpha = 180° - \gamma - ß = 79,4°$$

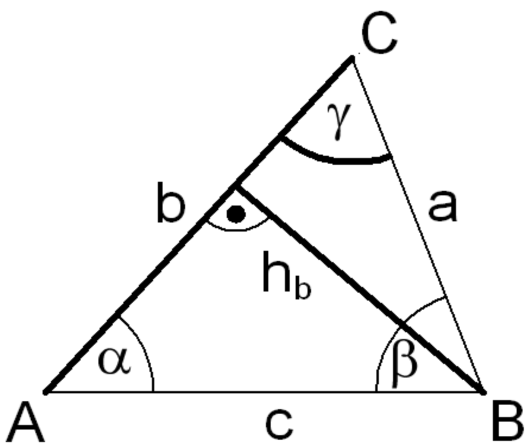
$$\frac{a}{b} = \frac{\sin\alpha}{\sin\gamma}$$

$$b = \frac{a \cdot \sin ß}{\sin\alpha} = 18,62\text{ cm}$$

$$\frac{a}{c} = \frac{\sin\alpha}{\sin\gamma}$$

$$c = \frac{a \cdot \sin\gamma}{\sin\alpha} = 11,5\text{ cm}$$

Lösung Aufgabe d)



$$\sin\gamma = \frac{h_b}{a}$$

$$a = \frac{h_b}{\sin\gamma} = 3,78\text{ cm}$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$$

$$c = 1,93\text{ cm}$$

$$\frac{a}{c} = \frac{\sin\alpha}{\sin\gamma}$$

$$\sin\alpha = \frac{a \cdot \sin\gamma}{c}$$

$$\alpha = 38,4°$$

$$ß = 180° - \alpha - \gamma = 123,1°$$

Lösung Aufgabe e)

$$\sin ß = \frac{h_a}{c}$$

$$ß = 65{,}5°$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos ß$$

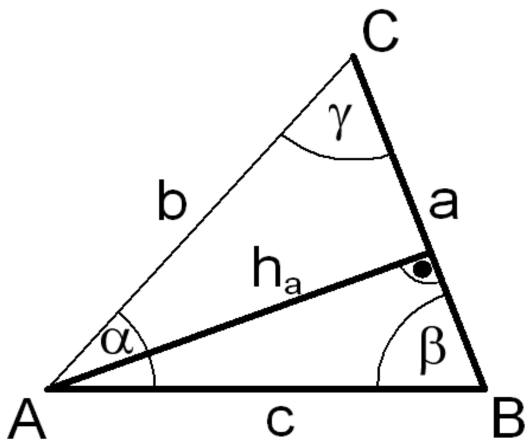
$$b = 9{,}88 \text{ cm}$$

$$\frac{a}{b} = \frac{\sin\alpha}{\sin ß}$$

$$\sin\alpha = \frac{a \cdot \sin ß}{b}$$

$$\alpha = 47{,}5°$$

$$\gamma = 180° - \alpha - ß = 67°$$



Lösung Aufgabe f)

$$s_b^{\,2} = a^2 + (0{,}5b)^2 - 2a \cdot 0{,}5b \cdot \cos\gamma$$

$$\cos\gamma = \frac{a^2 + (0{,}5b)^2 - s_b^{\,2}}{ab}$$

$$\gamma = 50{,}5°$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$$

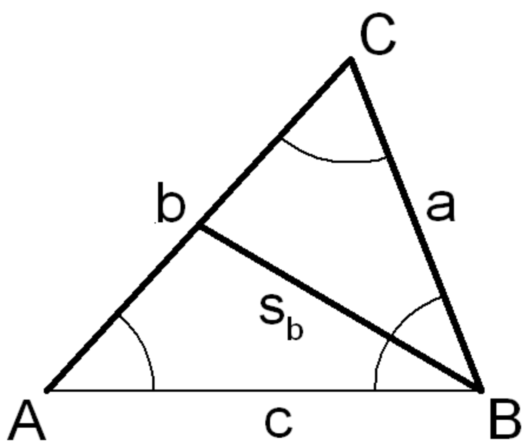
$$c = 3{,}1 \text{ cm}$$

$$\frac{a}{c} = \frac{\sin\alpha}{\sin\gamma}$$

$$\sin\alpha = \frac{a \cdot \sin\gamma}{c}$$

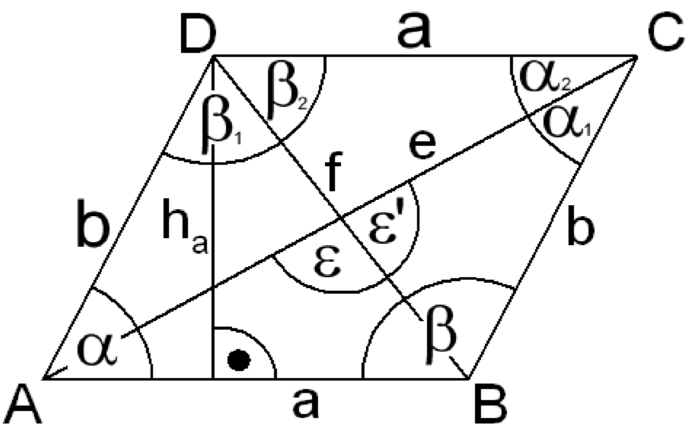
$$\alpha = 33{,}2°$$

$$ß = 180° - \alpha - \gamma = 96{,}3°$$



2. Berechne mit den gegebenen Werten des Parallelogramms ABCD die gesuchten Größen.

Gegeben:	**Gesucht:**
a) a = 17 cm; e = 23 cm; ß = 130°	α, b, f, A
b) b = 7,5 cm; f = 6,4 cm; α = 51°	ß, a, e, A
c) a = 3,25 cm; b = 2,75 cm; α = 75°	ß, f, A
d) e = 12,8 cm; f = 9,6 cm; ε = 112°	a, b, A
e) a = 5,4 cm; b = 3,8 cm; e = 7,48 cm	ß, α, f
f) a = 2,2 cm; b = 4 cm; f = 2,8 cm	b, α ß



Lösung Aufgabe a)

$\alpha = 180° - 130° = 50°$

$\frac{a}{e} = \frac{\sin\alpha_1}{\sin \text{ß}}$

$\sin\alpha_1 = \frac{a \cdot \sin \text{ß}}{e}$

$\alpha_1 = 34{,}5°$

$\alpha_2 = 50° - 34{,}5° = 15{,}5°$

$\frac{b}{e} = \frac{\sin\alpha_2}{\sin \text{ß}}$

$b = \frac{e \cdot \sin\alpha_2}{\sin \text{ß}} = 8{,}03\ \text{cm}$

$f^2 = a^2 + b^2 - 2ab \cdot \cos\alpha$

$f = 13{,}34\ \text{cm}$

$\sin\alpha = \frac{h_a}{b}$

$h_a = b \cdot \sin\alpha = 6{,}15\ \text{cm}$

$A = a \cdot h_a = 104{,}55\ \text{cm}^2$

Lösung Aufgabe b)

ß = 129°; $ß_2$ = 65,5°; $ß_1$ = 63,4°;
a= 7,36 cm; e = 13,41 cm;
h_a = 5,83 cm; A = 42,91 cm²

Lösung Aufgabe c)

ß = 105°; f = 3,67 cm; h_a= 2,66 cm;
A = 8,65 cm²

Lösung Aufgabe d)

a = 9,33 cm; ε' = 68°; b = 6,4 cm;
α = 72,5°; h_a = 6,1 cm; A = 56,91 cm²

Lösung Aufgabe e)

ß = 107,5°; α = 72,5°; f = 5,59 cm;

Lösung Aufgabe f)

ε = 91°; ε' = 89°; b = 2,26 cm;
α = 75,7°; ß = 104,3°